

# Woodturning and the 20-Ball Tetrahedral Pyramid



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## INTRODUCTION

Why this booklet on woodturning and the 20-ball tetrahedral pyramid puzzle?

At my local Woodturning Club, I have done various demonstrations and I try not to do the obvious such as bowls, hollow forms, etc, so when asked again I suggested combining two of my passions, turning and puzzles. This was accepted, and then I had to work out what I was going to turn for the demonstration. One of the items I decided on was the 20-ball pyramid consisting of four linear 3-ball units and two 4-ball units as being easy to turn. This has another benefit, as it provides good practice at turning beads and good tool control. To be ready for any questions, I started to research more into the puzzle and its many possibilities, and one thing led to another!

I purchased my first 20-ball pyramid puzzle in the mid- to late-nineteen seventies and it consisted of 4 pieces (shown here). This was the most common form of



the puzzle and has remained so over the intervening years, being marketed in various materials and sizes.

In his “Sources in Recreational Mathematics”, David Singmaster states that Piet Hein (of Soma Cube fame) created the first of these puzzles and it was marketed under the name of “Pyramystery” in 1970 – this is based on the copyright date in the leaflet supplied with the puzzle. Accompanying this was a fictitious story (or history!) of the origins of the puzzle. The original of this puzzle appears to have consisted of six pieces, two 4-ball linear and four 3-ball linear pieces. While another six-piece puzzle consisting of two 4-ball non-linear pieces, one 3-ball linear piece and three 3-ball non-linear pieces also appears to be attributed to Piet Hein and also called “Pyramystery”.

David Singmaster references several patents relating to Ball Pyramid puzzles and, using Free Patents Online, it is possible to see the whole patent and check any citations that they in turn reference.

US 3,837,652 Kuwagaki et al, Solid Puzzle, filed May 1, 1973, granted Sept. 24, 1974. They state that “the invention relates to a solid puzzle consisting of two or more spheres of the same diameter connected to each other by axes extending through the centers of the individual spheres, all of the said spheres being on the same plane”. They also state “one of the characteristics of the puzzle is that by assembling some of the connected units in a 3-dimensional space, a regular tetrahedron, regular quadrangular pyramid, equilateral triangular hexahedron or regular octahedron can be built”.

US 3,945,645 Roberts, Tangential Spheres Geometric Puzzle, filed Sept 23, 1974, granted Mar 23, 1976. This patent specifically describes a 35-ball puzzle comprising of 9 assembly pieces in a mix of planar and non-planar forms.

US 4,060,247 Kobres, Geometric Puzzle, filed June 28, 1976, granted Nov 29, 1977. This consists of five pieces, two of which each comprise of 6 connected balls in a planar rhomboid array, two pieces each comprised of two connected balls, and one piece of four connected balls in a planar linear array. The 5 pieces can be arranged either in a 20-ball 2-dimensional rhomboid or a 3 dimensional 20-ball tetrahedral pyramid.

US 4,988,103 Cheng, Geometric Puzzle of Spheres, filed Oct 2, 1989, granted Jan 29, 1991. This patent describes a fit-together type of puzzle comprising of seven 5-ball pieces which can be fitted together to form various shapes, one of which is a 35-ball tetrahedral pyramid.

US 6,637,745 B1 Vardanyan. Pyramid Puzzle System, filed Jan 22,2002, granted Oct 28, 2003. This consists of sets of puzzle pieces formed as identical spheres tangentially attached together in one or more row(s) on a square grid pattern. These pieces can be used to make pyramids. It shows a puzzle consisting of the 4 pieces diagrammed above and also expands the numbers of balls to make larger pyramids.

US D500,535S Thompson, Combined Baseball Stacking Puzzle and Pieces, filed Mar 27, 2003, granted Jan 4, 2005. Design consists of ten 2-ball pieces in the form of variously coloured baseballs, and the object is to arrange them into a 20-ball tetrahedral pyramid without the same colours touching.

On the Cubism for Fun website there are two papers of interest, one is “Some Notes on Ball-Pyramid and Related Puzzles (Revised 10 July 1986) written by Leonard Gordon of Chico, California, USA. This also contains some comments added by George Bell (also of USA) in 2013. The other paper is “Classification of Polyspheres” by George Bell.

David Singmaster also mentions Bernhard Wieszorke, of Dusseldorf, Germany, who published a 10-page booklet titled “Puzzling with Polyspheres” in Mar 1990. This was followed “Compendium of Polysphere Puzzles in 1995 with a 64-page Second Preliminary Edition in Aug 1996. I have not seen or been able to obtain either of these two references to date.

During the enforced lockdown due to the Coronavirus (Covid-19) pandemic in 2020, I decided that putting together a compendium of 20-ball tetrahedral pyramid pieces would be an interesting project, and this is the result.



# SECTION 1

## Puzzle Pieces, their Turning and Construction



# Potential 20-ball pyramid puzzle pieces

(Not all will be used in the enclosed puzzles)

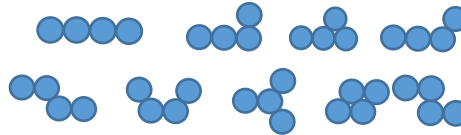
2 – ball pieces – Planar



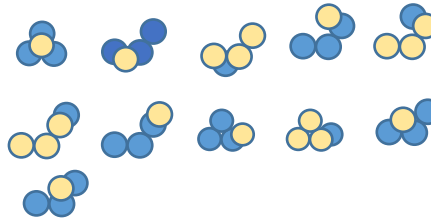
3 – ball pieces – Planar



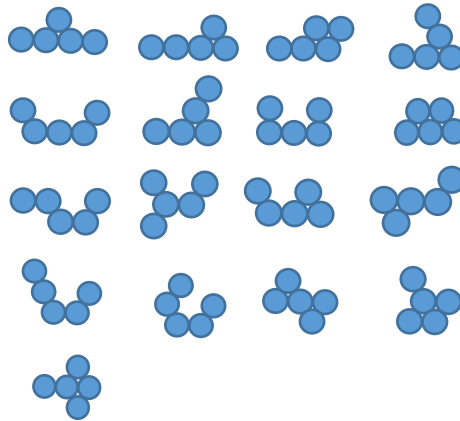
4-ball pieces – Planar



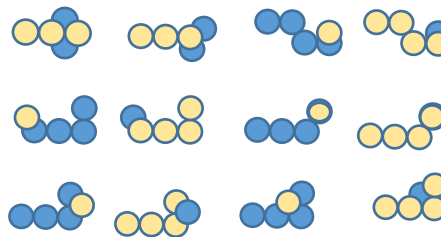
4-ball pieces – Non-planar



5-ball pieces – Planar



5-ball pieces – Non-planar



## NOTE



The yellow colour indicates a ball mounted on top of the blue balls in the position as drawn.

## **Making the Component Pieces on the Lathe**

For the puzzles shown, a ball of size 18mm diameter has been found to be both convenient to turn and to handle when attempting to solve the puzzles. Of course, the balls can be any diameter you may choose, for instance, I have one puzzle with balls as large as 50mm diameter and another with 8mm diameter balls.

The majority of the puzzle pieces require to be constructed from components of either single balls or strings of 2-, 3- or 4-balls. These various components will need to be glued together to form the individual puzzle pieces

The following pages show the process of turning a 3-ball component. This shows all you need to turn either a single ball or a 2-ball or 4-ball component.

When turning, it is important that you do so safely and wear appropriate protection for eyes and lungs, particularly important when sanding. Dust extraction is also recommended. The tools shown are those that I use, and any tools used for turning should be kept sharp – this not only means they cut better but safer as you do not have to force the tool to work.

An added bonus in making these pieces is that it is particularly good practice for both tool control and turning beads.



1. The tools used to turn the pieces are, from left to right, skew chisel, spindle gouge and a roughing out gouge.



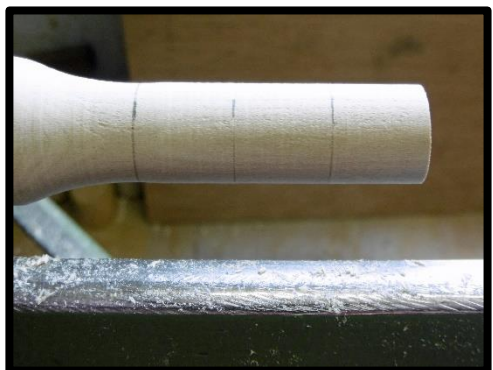
2. With the blank mounted in the 4-jaw chuck, use the roughing out gouge to reduce to the round along the length of the blank.



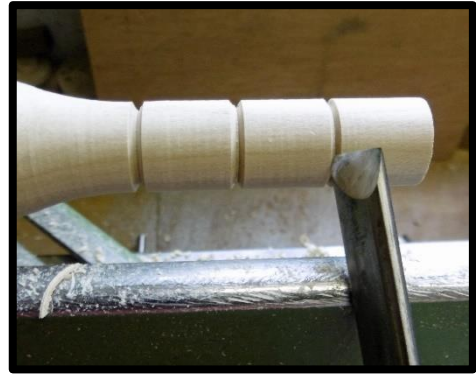
3. Continue with the roughing gouge and reduce the blank to 18mm in diameter. The skew chisel can be used to “fine tune” the diameter.



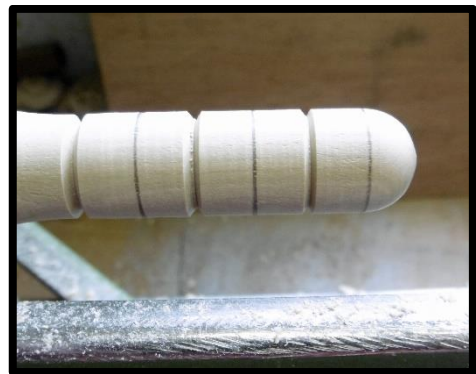
4. Square off the tailstock end of the blank with the skew chisel and mark off 18mm sections along the length.



5. Using the skew chisel cut V-grooves on the 18mm lines that have been marked out.



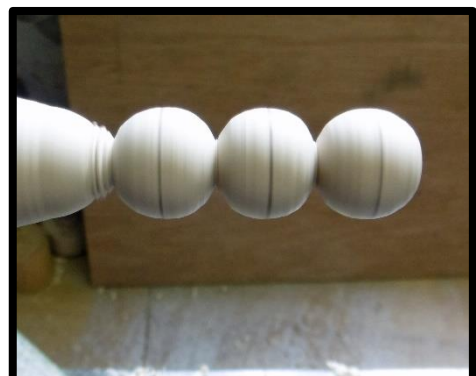
6. To aid turning the balls, put in centre lines between the 18mm grooves, then with the spindle gouge round over the end to form half a ball.



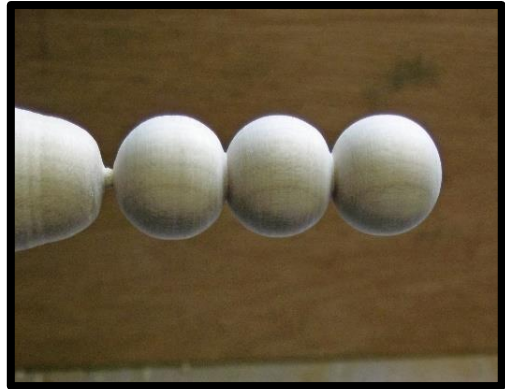
7. Now turn the second half of the ball, it is easier if the two halves of adjacent balls are turned together aiming to leave the centre lines visible.



8. Continue to form the balls in this way until the required number are complete. In this case, three.



9. Working through the grades (180 to 320 grit should be sufficient), lightly abrade the surface to remove tool marks and the centre mark, then part off the piece.



10. Hand sand away the small nub from parting off to give the completed piece. No finish should be required.



## Constructing the Puzzle Pieces

I have tried several adhesives to bond parts together to make the final pieces with varying success; I have tried cyanoacrylate, PVA, fast-curing epoxy and slow curing epoxy.

I have two types of cyanoacrylate adhesives, a thin viscosity one and a thicker viscosity version. Neither of them proved very suitable as it was not possible to keep the glue in a small bead long enough to position the parts. A benefit of these is that the cure is fast enough to allow the parts to be held in place in the hands. Possibly it was my technique but some of the pieces came apart after a short time. It is possible that a cyanoacrylate gel adhesive might work better but I have not tried it.

The PVA adhesive could be kept about in a small bead shape but was not ideal.

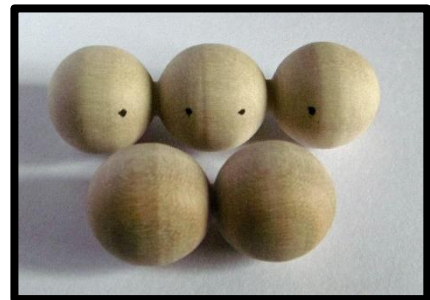
The fast-curing two-part epoxy was much better, forming beads and was sufficiently “tacky” to hold the parts together to allow them to be taped. The drawback was that the resin/hardener mix was curing too quickly to allow many pieces to be put together.

The two-part slow curing epoxy did allow time to make up multiple pieces and so is my preferred adhesive. The drawback is that you have to wait 24 hours for before the adhesive is fully cured. The following shows the process I use to glue the component parts together. If more than two parts are to be glued to make the final piece, then I generally glue them in separate stages, allowing the glue to harden between stages.

1. Take the parts to be joined and position them where they will be in the finished piece.



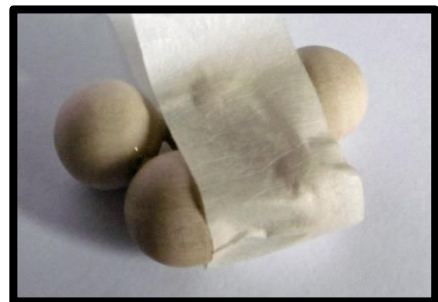
2. With the parts close together mark the points where they make contact. In this case there are four points; these indicate where the glue is to go.



3. A cocktail stick was used to place 4 small beads of epoxy onto the 4 marked points




















4. Bring the two parts together and use masking tape to hold them in close contact while the epoxy sets.






















5. The completed puzzle piece



**Table 1. Parts required for Construction of Puzzle Pieces**

Piece Shape	1-ball 	2-ball 	3-ball 	4-ball 
		X1		
			X1	
	X1	X1		
	X1	X1		
	X1	X1		
				X1
		X2		
		X2		
		X2		
		X2		
		X2		
	X2	X1		
		X2		

Piece Shape	1-ball	2-ball	3-ball	4-ball
				
		X2		
		X2		
	X1			X1
		X1	X1	
		X1	X1	
		X1	X1	
		X1	X1	
	X2		X1	
	X1	X2		
		X1	X1	
	X1	X2		
	X1	X2		
	X1	X2		
	X1	X2		














Piece Shape	1-ball	2-ball	3-ball	4-ball
				
		X1	X1	
		X1	X1	
	X1	X2		
	X1	X2		
	X2		X1	
			X2	
	X1		X1	
		X2		











The following Table 2 shows the various pieces required to make a complete set of pieces to accommodate all of the puzzles described in Section 2.

It shows the number of pieces required for each of the Planar 4-, Non-planar 4-, Planar 5-, Non-Planar5- and Planar 6-Piece puzzles described. In addition, the final column shows the various totals for the complete set.

**Table 2. Numbers of Pieces required for all the Puzzles described**

Piece Shape	Planar 4-Piece	Non-Planar 4-Piece	Planer 5-Piece	Non-Planer 5-Piece	Planar 6-Piece	Compendium
	X2					X2
	X2		X2		X2	X2
	X4	X1	X1			X4
	X2					X2
	X1	X1	X1			X1
	X2		X1			X2
	X4	X1				X4
	X1	X1	X2			X2
	X1		X1			X1
	X4					X4
	X4					X4
	X4					X4
		X1				X1

Piece Shape	Planar 4-Piece	Non-Planar 4-Piece	Planer 5-Piece	Non-Planer 5-Piece	Planar 6-Piece	Compendium
		X2				X2
		X2				X2
		X1				X1
		X1	X1			X1
			X5	X4	X1	X5
			X1		X1	X1
			X2			X2
			X2		X2	X2
			X2		X2	X2
			X1			X1
			X1			X1
			X1		X2	X2
			X1		X1	X1
	X2		X2			X2

Piece Shape	Planar 4-Piece	Non-Planar 4-Piece	Planer 5-Piece	Non-Planer 5-Piece	Planar 6-Piece	Compendium
	X2		X1		X4	X4
				X5		X5
				X2		X2
			X1		X4	X4
			X2		X1	X2
					X1	X1
	X2					X2
			X1			X1

## SECTION 2

### Various Puzzles and Solutions



This section gives various puzzles using the pieces described above. They are separated into Planar 4-, Non-planar 4-, Planar 5-, Non-Planar 5- and Planar 6-Piece puzzles. The puzzle pieces are numbered so that it is possible to see where they fit in each layer of the solution. The solution is presented as the four layers that make up the final tetrahedral pyramid. The first layer consists of 10 balls, the second 6 balls, the third 3 balls and the fourth layer 1 ball.

In some cases, they may be unique solutions but in others there may be more than one solution; I have chosen to show one solution only. A computer program would be useful to check all possible solutions, however I do not have access to one and these solutions have been arrived at by playing with the puzzle. Remember, you may think you have found a different solution to the one shown – check that it is not just a rotation or reflection!

Some of the puzzles are trivial and easy to solve but they are included for completeness and the fun.

**NOTE:** Several of these puzzle designs are commercially available and, as such, will be covered by copyright law. There is nothing to stop you making them for your own use, but they should not be made for sale.

## SECTION 2.A 4-Piece Puzzles using only Planar Pieces

	Layers				Pieces			
1.	3 44 222 1111	3 44 222 44	3 44 3 44	3 3	1	2	3	4
2.	1 11 112 3332	4 42 332	4 42 42	4 4	1	2	3	4
3.	4 22 131 3333	4 24 111	4 4 22	4 4	1	2	3	4
4.	4 41 111 2221	4 43 223	3 43	3 3	1	2	3	4
5.	2 21 221 3233	4 41 331	4 4 41	4 4	1	2	3	4
6.	1 12 112 1332	4 42 433	2 43	4	1	2	3	4
7.	1 21 211 2413	2 23 443	3 43	4	1	2	3	4
8.	2 31 112 3411	2 32 344	3 42	4	1	2	3	4

9.  $\begin{matrix} 3 \\ 34 & 4 \\ 111 & 34 & 4 \\ 1212 & 333 & 23 & 2 \end{matrix}$   $\begin{matrix} 1 & 2 & 3 & 4 \\ \text{[Diagram 1]} & \text{[Diagram 2]} & \text{[Diagram 3]} & \text{[Diagram 4]} \end{matrix}$

10.  $\begin{matrix} 1 \\ 21 & 4 \\ 214 & 24 & 4 \\ 3114 & 333 & 23 & 2 \end{matrix}$   $\begin{matrix} 1 & 2 & 3 & 4 \\ \text{[Diagram 1]} & \text{[Diagram 2]} & \text{[Diagram 3]} & \text{[Diagram 4]} \end{matrix}$

11.  $\begin{matrix} 1 \\ 31 & 2 \\ 312 & 32 & 2 \\ 3112 & 344 & 44 & 4 \end{matrix}$   $\begin{matrix} 1 & 2 & 3 & 4 \\ \text{[Diagram 1]} & \text{[Diagram 2]} & \text{[Diagram 3]} & \text{[Diagram 4]} \end{matrix}$

12.  $\begin{matrix} 3 \\ 44 & 3 \\ 111 & 44 & 3 \\ 1212 & 222 & 44 & 3 \end{matrix}$   $\begin{matrix} 1 & 2 & 3 & 4 \\ \text{[Diagram 1]} & \text{[Diagram 2]} & \text{[Diagram 3]} & \text{[Diagram 4]} \end{matrix}$

13.  $\begin{matrix} 1 \\ 22 & 1 \\ 233 & 22 & 1 \\ 3344 & 444 & 22 & 1 \end{matrix}$   $\begin{matrix} 1 & 2 & 3 & 4 \\ \text{[Diagram 1]} & \text{[Diagram 2]} & \text{[Diagram 3]} & \text{[Diagram 4]} \end{matrix}$

14.  $\begin{matrix} 3 \\ 34 & 4 \\ 111 & 34 & 3 \\ 1122 & 222 & 34 & 4 \end{matrix}$   $\begin{matrix} 1 & 2 & 3 & 4 \\ \text{[Diagram 1]} & \text{[Diagram 2]} & \text{[Diagram 3]} & \text{[Diagram 4]} \end{matrix}$

15.  $\begin{matrix} 1 \\ 44 & 1 \\ 333 & 44 & 1 \\ 2222 & 323 & 44 & 1 \end{matrix}$   $\begin{matrix} 1 & 2 & 3 & 4 \\ \text{[Diagram 1]} & \text{[Diagram 2]} & \text{[Diagram 3]} & \text{[Diagram 4]} \end{matrix}$





16.  $\begin{matrix} 1 \\ 14 & 4 \\ 333 & 14 & 1 \\ 2222 & 323 & 14 & 4 \end{matrix}$   $\begin{matrix} 1 & 2 & 3 & 4 \\ \text{[Diagram 1]} & \text{[Diagram 2]} & \text{[Diagram 3]} & \text{[Diagram 4]} \end{matrix}$

17.  $\begin{matrix} 1 \\ 14 & 1 \\ 333 & 14 & 4 \\ 2222 & 323 & 14 & 4 \end{matrix}$   $\begin{matrix} 1 & 2 & 3 & 4 \\ \text{[Diagram 1]} & \text{[Diagram 2]} & \text{[Diagram 3]} & \text{[Diagram 4]} \end{matrix}$



18.

1			
14	3		
314	34	4	
2211	324	32	2

1	2	3	4
			


---

## SECTION 2.B 4-Piece Puzzles using both Planar and Non-Planar Pieces

1.

1			
21	1		
233	24	1	
3334	244	24	1

1      2      3      4




---

2.

1			
22	1		
433	21	2	
3344	443	11	2

1      2      3      4




---

3.

1			
22	1		
333	12	2	
4433	444	11	2

1      2      3      4




---

4.

2			
23	2		
333	24	2	
1113	144	14	4

1      2      3      4




---

5.

1			
11	4		
121	44	4	
2222	333	33	4

1      2      3      4




---

6.

1			
22	1		
333	21	2	
4343	444	11	2

1      2      3      4




---

7.

2			
21	2		
113	43	2	
1143	443	43	2

1      2      3      4



---

8.

1			
11	2		
333	21	2	
4343	444	21	2

1	2	3	4

---

9.

1			
11	2		
434	21	2	
3333	444	21	2

1	2	3	4

---

10.

1			
11	2		
444	21	2	
3333	444	21	2

1	2	3	4

---

11.

1			
11	2		
333	21	2	
3344	444	21	2

1	2	3	4

---

12.

1			
11	2		
334	21	2	
4433	344	21	2

1	2	3	4

---

13.

1			
11	2		
433	21	2	
3344	443	21	2

1	2	3	4

---

14.

1			
22	1		
434	12	2	
3333	444	11	2

1	2	3	4

---

15.

1			
22	1		
444	12	2	
4343	333	11	2

1	2	3	4

---

16.





1			
22	1		
444	12	2	
3344	333	11	2

1	2	3	4

---

17.





1			
22	1		
444	12	2	
3333	444	11	2

1	2	3	4
			

---

18.





1			
22	1		
333	21	2	
4433	444	11	2

1	2	3	4
			

---

19.





1			
22	1		
434	21	2	
3333	444	11	2

1	2	3	4
			

---

20.





1			
22	1		
444	21	2	
3333	444	11	2

1	2	3	4
			

---

21.

1			
22	1		
333	21	2	
4343	444	11	2

1	2	3	4
			

---

## SECTION 2.C 5-Piece Puzzles using only Planar Pieces

1.

2				
12	5			
132	12	5		
1333	444	54	5	

1	2	3	4	5

---

2.

2				
34	2			
555	34	2		
1111	345	34	2	

1	2	3	4	5

---

3.

2				
22	5			
213	53	5		
1113	544	43	4	

1	2	3	4	5

---

4.

3				
22	3			
224	34	4		
1111	354	55	5	

1	2	3	4	5

---

5.

2				
22	4			
421	43	3		
1113	553	45	5	

1	2	3	4	5

---

6.

1				
11	4			
121	34	4		
2222	333	55	5	

1	2	3	4	5

---

7.

1				
11	2			
131	24	4		
3333	222	54	5	

1	2	3	4	5

---

8.

4				
41	4			
431	51	5		
3331	222	21	2	

1	2	3	4	5

---

9.  $\begin{matrix} 4 \\ 34 & 1 \\ 334 & 21 & 5 \\ 3344 & 211 & 25 & 2 \end{matrix}$       1      2      3      4      5

---

10.  $\begin{matrix} 2 \\ 24 & 3 \\ 322 & 34 & 4 \\ 1111 & 555 & 35 & 4 \end{matrix}$       1      2      3      4      5

---

11.  $\begin{matrix} 1 \\ 15 & 3 \\ 315 & 34 & 4 \\ 2211 & 325 & 42 & 2 \end{matrix}$       1      2      3      4      5

---

12.  $\begin{matrix} 1 \\ 13 & 5 \\ 413 & 53 & 5 \\ 2211 & 423 & 42 & 2 \end{matrix}$       1      2      3      4      5

---

13.  $\begin{matrix} 2 \\ 23 & 2 \\ 333 & 25 & 2 \\ 1113 & 154 & 14 & 4 \end{matrix}$       1      2      3      4      5

---

14.  $\begin{matrix} 3 \\ 13 & 4 \\ 213 & 24 & 5 \\ 2111 & 444 & 25 & 2 \end{matrix}$       1      2      3      4      5

---

15.  $\begin{matrix} 1 \\ 14 & 3 \\ 114 & 34 & 4 \\ 5555 & 332 & 22 & 2 \end{matrix}$       1      2      3      4      5

---

16.  $\begin{matrix} 2 \\ 42 & 3 \\ 223 & 43 & 4 \\ 1111 & 553 & 55 & 4 \end{matrix}$       1      2      3      4      5

---

17.  $\begin{matrix} 2 \\ 21 & 5 \\ 221 & 55 & 4 \\ 4113 & 453 & 43 & 3 \end{matrix}$       1      2      3      4      5

---

18.  $\begin{matrix} 3 \\ 31 & 3 \\ 115 & 35 & 5 \\ 1224 & 244 & 24 & 5 \end{matrix}$   $\begin{matrix} 1 & 2 & 3 & 4 & 5 \\ \text{[Diagram 1]} & \text{[Diagram 2]} & \text{[Diagram 3]} & \text{[Diagram 4]} & \text{[Diagram 5]} \end{matrix}$

19.  $\begin{matrix} 1 \\ 12 & 4 \\ 122 & 54 & 5 \\ 1234 & 334 & 53 & 5 \end{matrix}$   $\begin{matrix} 1 & 2 & 3 & 4 & 5 \\ \text{[Diagram 1]} & \text{[Diagram 2]} & \text{[Diagram 3]} & \text{[Diagram 4]} & \text{[Diagram 5]} \end{matrix}$

20.  $\begin{matrix} 2 \\ 22 & 3 \\ 324 & 34 & 4 \\ 5111 & 551 & 35 & 4 \end{matrix}$   $\begin{matrix} 1 & 2 & 3 & 4 & 5 \\ \text{[Diagram 1]} & \text{[Diagram 2]} & \text{[Diagram 3]} & \text{[Diagram 4]} & \text{[Diagram 5]} \end{matrix}$

21.  $\begin{matrix} 1 \\ 41 & 4 \\ 221 & 45 & 5 \\ 2321 & 433 & 35 & 5 \end{matrix}$   $\begin{matrix} 1 & 2 & 3 & 4 & 5 \\ \text{[Diagram 1]} & \text{[Diagram 2]} & \text{[Diagram 3]} & \text{[Diagram 4]} & \text{[Diagram 5]} \end{matrix}$

22.  $\begin{matrix} 4 \\ 14 & 5 \\ 311 & 34 & 5 \\ 3122 & 324 & 52 & 5 \end{matrix}$   $\begin{matrix} 1 & 2 & 3 & 4 & 5 \\ \text{[Diagram 1]} & \text{[Diagram 2]} & \text{[Diagram 3]} & \text{[Diagram 4]} & \text{[Diagram 5]} \end{matrix}$

23.  $\begin{matrix} 1 \\ 11 & 4 \\ 312 & 34 & 4 \\ 5222 & 534 & 53 & 5 \end{matrix}$   $\begin{matrix} 1 & 2 & 3 & 4 & 5 \\ \text{[Diagram 1]} & \text{[Diagram 2]} & \text{[Diagram 3]} & \text{[Diagram 4]} & \text{[Diagram 5]} \end{matrix}$

24.  $\begin{matrix} 4 \\ 24 & 4 \\ 122 & 15 & 4 \\ 1233 & 335 & 15 & 5 \end{matrix}$   $\begin{matrix} 1 & 2 & 3 & 4 & 5 \\ \text{[Diagram 1]} & \text{[Diagram 2]} & \text{[Diagram 3]} & \text{[Diagram 4]} & \text{[Diagram 5]} \end{matrix}$

25.  $\begin{matrix} 2 \\ 22 & 5 \\ 213 & 43 & 5 \\ 1113 & 444 & 53 & 5 \end{matrix}$   $\begin{matrix} 1 & 2 & 3 & 4 & 5 \\ \text{[Diagram 1]} & \text{[Diagram 2]} & \text{[Diagram 3]} & \text{[Diagram 4]} & \text{[Diagram 5]} \end{matrix}$

26.  $\begin{matrix} 1 \\ 21 & 4 \\ 214 & 54 & 5 \\ 2133 & 234 & 53 & 5 \end{matrix}$   $\begin{matrix} 1 & 2 & 3 & 4 & 5 \\ \text{[Diagram 1]} & \text{[Diagram 2]} & \text{[Diagram 3]} & \text{[Diagram 4]} & \text{[Diagram 5]} \end{matrix}$

27.  $\begin{matrix} 3 \\ 32 & 5 \\ 222 & 35 & 5 \\ 1114 & 144 & 34 & 5 \end{matrix}$       1      2      3      4      5

28.  $\begin{matrix} 2 \\ 22 & 4 \\ 213 & 44 & 5 \\ 1113 & 543 & 53 & 5 \end{matrix}$       1      2      3      4      5

29.  $\begin{matrix} 3 \\ 32 & 5 \\ 332 & 44 & 5 \\ 1122 & 414 & 51 & 5 \end{matrix}$       1      2      3      4      5

30.  $\begin{matrix} 3 \\ 33 & 4 \\ 113 & 45 & 4 \\ 1122 & 425 & 25 & 5 \end{matrix}$       1      2      3      4      5

31.  $\begin{matrix} 4 \\ 22 & 4 \\ 112 & 54 & 4 \\ 1123 & 533 & 53 & 5 \end{matrix}$       1      2      3      4      5

32.  $\begin{matrix} 2 \\ 11 & 2 \\ 135 & 25 & 5 \\ 1333 & 254 & 44 & 4 \end{matrix}$       1      2      3      4      5

33.  $\begin{matrix} 2 \\ 11 & 2 \\ 134 & 24 & 4 \\ 1333 & 254 & 55 & 5 \end{matrix}$       1      2      3      4      5

34.  $\begin{matrix} 2 \\ 11 & 2 \\ 135 & 25 & 5 \\ 1333 & 244 & 54 & 4 \end{matrix}$       1      2      3      4      5

35.  $\begin{matrix} 4 \\ 44 & 2 \\ 143 & 23 & 3 \\ 2111 & 253 & 55 & 5 \end{matrix}$       1      2      3      4      5



36. 
$$\begin{array}{r} 1 \\ 13 \quad 3 \\ 213 \quad 24 \quad 4 \\ 2143 \quad 245 \quad 55 \quad 5 \end{array}$$

1 2 3 4 5

---

37. 
$$\begin{array}{r} 5 \\ 53 \quad 3 \\ 155 \quad 34 \quad 4 \\ 1112 \quad 322 \quad 24 \quad 4 \end{array}$$

1 2 3 4 5

---

38. 
$$\begin{array}{r} 2 \\ 52 \quad 5 \\ 122 \quad 45 \quad 4 \\ 3111 \quad 335 \quad 43 \quad 4 \end{array}$$

1 2 3 4 5

---

39. 
$$\begin{array}{r} 4 \\ 22 \quad 4 \\ 352 \quad 41 \quad 1 \\ 3332 \quad 555 \quad 41 \quad 1 \end{array}$$

1 2 3 4 5

---

40. 
$$\begin{array}{r} 2 \\ 52 \quad 5 \\ 342 \quad 35 \quad 3 \\ 4442 \quad 115 \quad 11 \quad 3 \end{array}$$

1 2 3 4 5

---

41. 
$$\begin{array}{r} 2 \\ 21 \quad 4 \\ 213 \quad 45 \quad 4 \\ 1153 \quad 253 \quad 53 \quad 4 \end{array}$$

1 2 3 4 5

---

42. 
$$\begin{array}{r} 3 \\ 33 \quad 4 \\ 435 \quad 41 \quad 1 \\ 4555 \quad 221 \quad 22 \quad 1 \end{array}$$

1 2 3 4 5

---

43. 
$$\begin{array}{r} 5 \\ 43 \quad 5 \\ 113 \quad 43 \quad 5 \\ 2211 \quad 223 \quad 44 \quad 5 \end{array}$$

1 2 3 4 5

---

44. 
$$\begin{array}{r} 4 \\ 43 \quad 5 \\ 113 \quad 43 \quad 5 \\ 1122 \quad 223 \quad 45 \quad 5 \end{array}$$

1 2 3 4 5

---

45.  $\begin{matrix} 1 \\ 31 & 4 \\ 112 & 34 & 3 \\ 2224 & 354 & 55 & 5 \end{matrix}$

1 2 3 4 5

---

46.  $\begin{matrix} 3 \\ 12 & 3 \\ 112 & 35 & 5 \\ 1224 & 444 & 45 & 5 \end{matrix}$

1 2 3 4 5

---

47.  $\begin{matrix} 3 \\ 22 & 3 \\ 224 & 34 & 5 \\ 1111 & 345 & 45 & 5 \end{matrix}$

1 2 3 4 5

---

48.  $\begin{matrix} 2 \\ 22 & 4 \\ 211 & 45 & 5 \\ 3311 & 433 & 45 & 5 \end{matrix}$

1 2 3 4 5

---

49.  $\begin{matrix} 3 \\ 13 & 3 \\ 211 & 25 & 3 \\ 2144 & 245 & 45 & 5 \end{matrix}$

1 2 3 4 5

---

## SECTION 2.D 5-Piece Puzzles using both Planar and Non-Planar Pieces

1.	3					1	2	3	4	5
	33	3								
	142	44	5							
	1122	142	55	5		<hr/>				
2.	3					1	2	3	4	5
	32	3								
	312	42	5							
	1112	444	55	5		<hr/>				
3.	1					1	2	3	4	5
	21	5								
	211	54	5							
	2244	354	33	3		<hr/>				
4.	1					1	2	3	4	5
	14	3								
	314	55	5							
	2211	324	32	2		<hr/>				
5.	3					1	2	3	4	5
	11	3								
	114	34	5							
	2224	324	55	5		<hr/>				
6.	2					1	2	3	4	5
	11	2								
	431	42	5							
	3331	452	45	5		<hr/>				

## SECTION 2.E 6-Piece Puzzles using only Planar Pieces

1.

6			
45	6		
222	45	6	
1111	333	45	6

1	2	3	4	5	6
---	---	---	---	---	---

---

2.

6			
45	6		
222	45	6	
1111	333	45	6

1	2	3	4	5	6
---	---	---	---	---	---

---

3.

4			
42	4		
533	42	6	
5533	111	62	6

1	2	3	4	5	6
---	---	---	---	---	---

---

4.

6			
44	6		
222	45	6	
1111	333	45	6

1	2	3	4	5	6
---	---	---	---	---	---

---

5.

2			
22	6		
113	33	6	
1144	554	54	6

1	2	3	4	5	6
---	---	---	---	---	---

---

6.

1			
44	1		
556	46	6	
3333	562	42	2

1	2	3	4	5	6
---	---	---	---	---	---

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## **APPENDIX**

This Section contains copies of the front pages of the patents referenced in the Introduction

- [54] **SOLID PUZZLE**
- [75] Inventors: **Akira Kuwagaki; Sadao Takenaka,**  
both of Kyoto, Japan
- [73] Assignee: **Nippon Cloth Industry Co. Ltd.,**  
Kyoto, Japan
- [22] Filed: **May 1, 1973**
- [21] Appl. No.: **356,227**

[30] **Foreign Application Priority Data**  
May 13, 1972 Japan..... 47-46886

- [52] U.S. Cl..... **273/157 R**
- [51] Int. Cl. .... **A63f 9/12**
- [58] Field of Search ..... **273/157, 146; 35/18 A,**  
**35/34, 72**

- [56] **References Cited**  
**UNITED STATES PATENTS**
- |           |        |                 |            |
|-----------|--------|-----------------|------------|
| 2,151,030 | 3/1939 | Hinsen .....    | 273/146 UX |
| 2,836,421 | 5/1958 | Turner .....    | 273/160    |
| 2,942,356 | 6/1960 | Weintraub ..... | 35/18 A    |

3,091,870 6/1963 Sangster..... 35/18 A X

**FOREIGN PATENTS OR APPLICATIONS**

52,215 9/1936 Denmark ..... 273/157 R

**OTHER PUBLICATIONS**

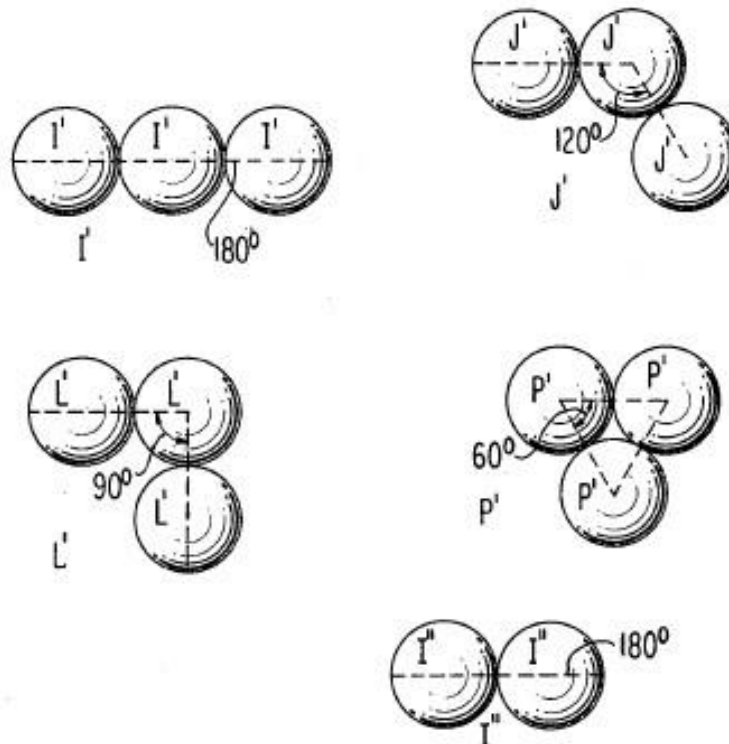
Martin Gardner's New Mathematical Diversions from Scientific American, publ. by Simon & Schuster, New York, 1966, pages 82-90.

*Primary Examiner*—Anton O. Oechsle  
*Attorney, Agent, or Firm*—Sughrue, Rothwell, Mion, Zinn & Macpeak

[57] **ABSTRACT**

A solid puzzle of regular geometrical configuration consists of a number of layers of identical spheres, each of said layers including several kinds of connected units of spheres having connecting axes extending through the centers of the individual spheres, all of the connecting axes being on the same plane and any of two adjoining axes forming an included angle of 60°, 90°, 120° or 180°.

**2 Claims, 18 Drawing Figures**



[54] TANGENTIAL SPHERES GEOMETRIC PUZZLE

[76] Inventor: Patrick A. Roberts, 4208 Jefferson St., Hyattsville, Md. 20781

[22] Filed: Sept. 23, 1974

[21] Appl. No.: 508,635

[52] U.S. Cl. .... 273/157 R

[51] Int. Cl.<sup>2</sup> ..... A63F 9/12

[58] Field of Search ..... 273/157 R, 146; 35/18 A, 35/34, 72

[56] References Cited UNITED STATES PATENTS

1,709,660	4/1929	De Bracht	273/157 R
2,942,356	6/1960	Weintraub	35/18 A
3,091,870	6/1963	Sangster	35/18 A X
3,837,652	9/1974	Kuwagaki et al.	273/157 R

FOREIGN PATENTS OR APPLICATIONS

52,215	9/1936	Denmark	273/157 R
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OTHER PUBLICATIONS

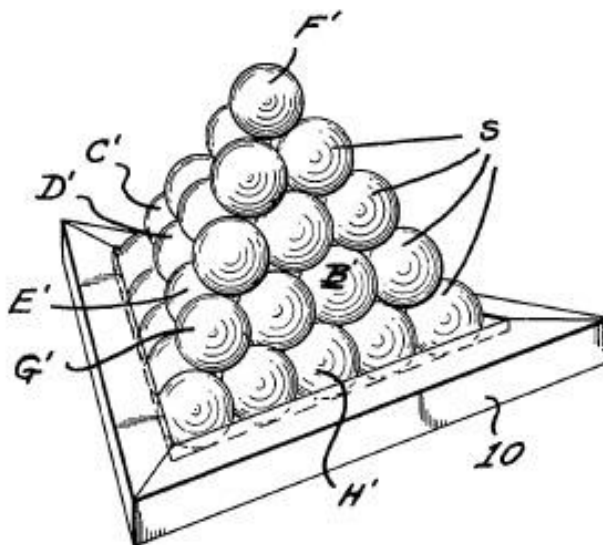
Martin Gardner's New Mathematical Diversions from Scientific American, publ. by Simon & Schuster, New York, 1966, pp. 82-90.

Primary Examiner—Anton O. Oechsle  
Attorney, Agent, or Firm—Low & Matthews

[57] ABSTRACT

A tangential spheres geometric puzzle uniquely formed from nine assembly pieces, of which eight pieces are based upon a triangular sphere array and a ninth piece based upon spheres disposed in a square array, providing a challenging educational and manipulative game. The pieces are assemblable into a tetrahedral pyramid.

8 Claims, 20 Drawing Figures



**United States Patent** [19]

[11] **4,060,247**

**Kobres, Jr.**

[45] **Nov. 29, 1977**

[54] **GEOMETRIC PUZZLE**

[76] Inventor: **Robert E. Kobres, Jr., Rte. 1,  
Nicholson, Ga. 30565**

[21] Appl. No.: **681,519**

[22] Filed: **June 28, 1976**

[51] Int. Cl.<sup>2</sup> ..... **A63F 9/12**

[52] U.S. Cl. .... **273/157 R; 35/34**

[58] Field of Search ..... **273/157 R; 35/18 A,  
35/34**

[56] **References Cited**

**U.S. PATENT DOCUMENTS**

3,837,652 9/1972 **Kuwagaki et al.** ..... 273/157 R  
3,945,645 3/1976 **Roberts** ..... 273/157 R

**OTHER PUBLICATIONS**

**Martin Gardner's New Mathematical Diversions from**

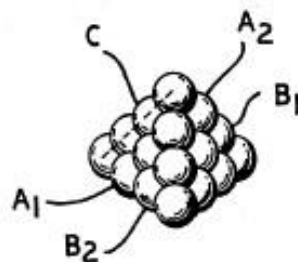
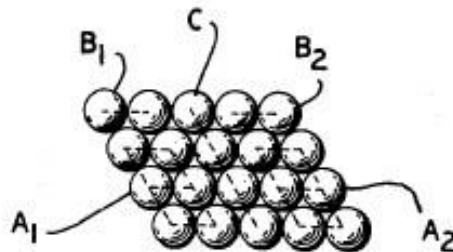
**Scientific American**, published by **Simon & Schuster**,  
**New York 1966**, pp. 82-90.

*Primary Examiner*—**Anton O. Oechsle**  
*Attorney, Agent, or Firm*—**Frank A. Peacock**

[57] **ABSTRACT**

A five-piece geometric puzzle is disclosed. This geometric puzzle uses three basic shapes which can be arranged either as a two dimensional rhomboid or a three dimensional tetrahedron. The puzzle set is constituted by two pieces each comprising six connected balls in a planar rhomboid array, two pieces each comprising two connected balls, and one piece comprising four connected balls in planar linear array. Multiple sets of the pieces may be provided.

**2 Claims, 10 Drawing Figures**





- [54] **GEOMETRIC PUZZLE OF SPHERES**  
 [76] **Inventor:** Ming S. Cheng, 1 FL, No. 12, Alley 20, Lane 66, Tung Yuang St., Taipei, Taiwan  
 [21] **Appl. No.:** 415,962  
 [22] **Filed:** Oct. 2, 1989  
 [51] **Int. Cl.<sup>5</sup>** ..... A63F 9/12  
 [52] **U.S. Cl.** ..... 273/157 R  
 [58] **Field of Search** ..... 273/153 R, 153 P, 157; 434/211, 213, 277, 278

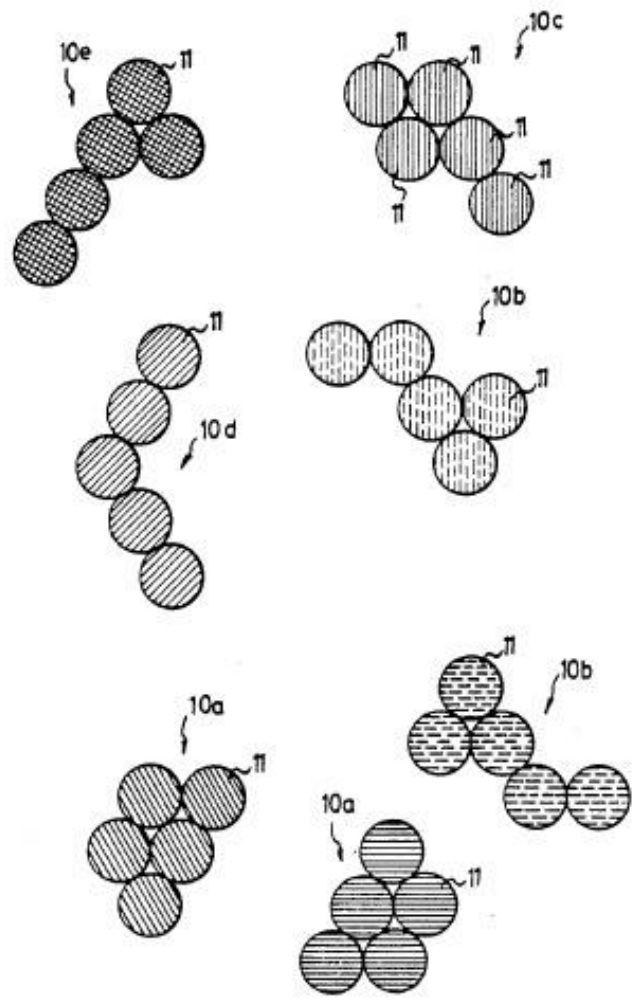
[56] **References Cited**  
**U.S. PATENT DOCUMENTS**  
 3,837,652 9/1974 Kuwagaki et al. .... 273/157 R  
 3,945,645 3/1976 Roberts ..... 273/157 R  
 4,060,247 11/1977 Kobres ..... 273/157 R  
 4,650,424 3/1987 Mitchell ..... 434/211  
 4,699,602 10/1987 Giorgi ..... 434/211

*Primary Examiner*—Edward M. Coven

*Assistant Examiner*—W. Pierce  
*Attorney, Agent, or Firm*—Morton J. Rosenberg; David I. Klein

[57] **ABSTRACT**  
 A fit-together type puzzle comprising a number of ball blocks of varying configurations with each ball block being composed of an equal number of balls joined together. The ball blocks are fitted together with one another to selectably form a planar triangular figure, a planar elongate hexagonal figure, and a solid regular tetrahedral figure. Each ball block is composed of such number of balls that the planar triangular figure is composed of thirty-five balls, the planar elongate hexagonal figure is composed of thirty balls, and the solid regular tetrahedral figure has one side thereof composed of fifteen balls. A board which has recesses or a set of semi-spherical depressions thereon corresponding to the figures is provided to receive the balls blocks.

3 Claims, 5 Drawing Sheets





US006637745B1

(12) **United States Patent**  
**Vardanyan**

(10) **Patent No.:** **US 6,637,745 B1**  
(45) **Date of Patent:** **Oct. 28, 2003**

(54) **PYRAMID PUZZLE SYSTEM**

(76) Inventor: **Gevorg Vardanyan**, 412 N. Jackson St., #101, Glendale, CA (US) 91206

(\* ) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 0 days.

(21) Appl. No.: **10/051,687**

(22) Filed: **Jan. 22, 2002**

(51) **Int. Cl.**<sup>7</sup> ..... **A63F 9/12**

(52) **U.S. Cl.** ..... **273/153 P; 273/153 R**

(58) **Field of Search** ..... **273/156, 153 P, 273/153 R; 434/236, 211**

(56) **References Cited**

**U.S. PATENT DOCUMENTS**

3,837,652 A	*	9/1974	Kuwagaki et al.	.....	273/157 R
3,945,645 A	*	3/1976	Roberts	.....	273/157 R
4,060,247 A	*	11/1977	Kobres, Jr.	.....	273/157 R
4,988,103 A	*	1/1991	Cheng	.....	273/157 R
5,988,640 A	*	11/1999	Wheeler	.....	273/24

\* cited by examiner 1

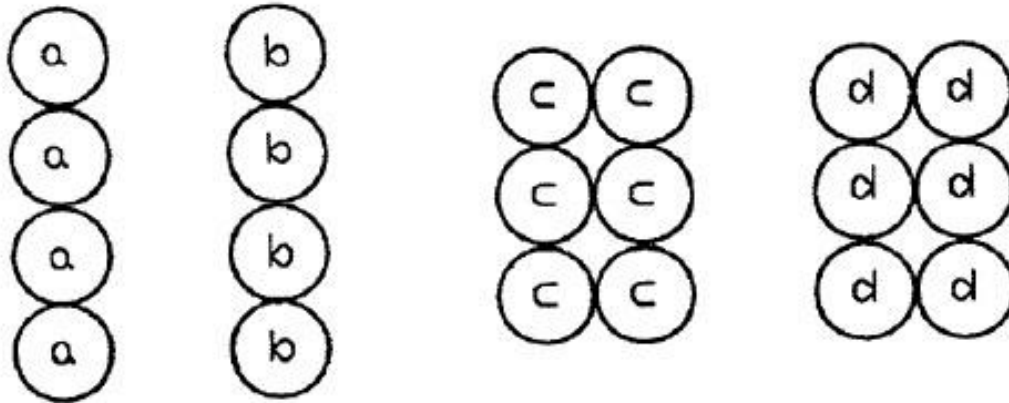
*Primary Examiner*—Kien T. Nguyen

(74) *Attorney, Agent, or Firm*—J. E. McTaggart

(57) **ABSTRACT**

A system of pyramid puzzles provides sets of puzzle-pieces formed as identical spheres tangentially attached together in one or more row(s) on a square grid pattern, configured to provide interesting challenges with surprising solutions in assembling pyramids. The puzzle system includes two families of sets of matched pairs of puzzle pieces for assembling pyramids of two corresponding equilateral shapes: triangle-based and square-based. For the triangular-based pyramid, puzzle-pieces are configured from single or plural attached rows of spheres forming rectangular outlines that each form an inclined layer in the pyramid. For the square-based pyramid, puzzle-pieces are configured in generally L-shaped orthogonal forms in matched pairs that each combine to form a horizontal layer in the pyramid. The sets can range from relatively simple with as few as four puzzle-pieces to larger sets of any desired size, increasing in progressive steps, in accordance with the teachings of the invention.

**18 Claims, 5 Drawing Sheets**





US00D500535S

(12) **United States Design Patent** (10) **Patent No.:** **US D500,535 S**  
**Thompson** (45) **Date of Patent:** **\*\* Jan. 4, 2005**

(54) **COMBINED BASEBALL STACKING PUZZLE AND PIECES**

(76) Inventor: **Raymond B. Thompson, 2756 Marshall St., Falls Church, VA (US) 22042**

(\*\*) Term: **14 Years**

(21) Appl. No.: **29/178,429**

(22) Filed: **Mar. 27, 2003**

**Related U.S. Application Data**

(63) Continuation-in-part of application No. 29/175,365, filed on Feb. 5, 2003.

(51) **LOC (7) Cl.** ..... **21-01**

(52) **U.S. Cl.** ..... **D21/478**

(58) **Field of Search** ..... **D21/300, 301, D21/314-319, 471, 478, 479, 386, 707-714; D9/337, 338, 341, 307, 418, 441, 451; D11/131, 157; D19/42, 97; D7/651, 654; D3/211; 273/153 R, 156, 157 R, 153 P, 146, 241, 258-260, 264, 271, 276, 288, 290; 206/579, 315.1, 315.9, 775, 776, 781, 557, 564**

(56) **References Cited**

**U.S. PATENT DOCUMENTS**

2,151,030	A	*	3/1939	Hinsen	.....	273/146	X
3,624,939	A	*	12/1971	Gossard	.....	D9/307	X
3,837,652	A		9/1974	Kuwagaki et al.			
3,945,645	A		3/1976	Roberts			
4,057,247	A		11/1977	Morrison			
4,060,247	A		11/1977	Kobres, Jr.			
4,325,552	A		4/1982	Glasheen			
4,988,103	A		1/1991	Cheng			
5,381,895	A	*	1/1995	Thomsen	.....	206/315.9	

(List continued on next page.)

**OTHER PUBLICATIONS**

Tetra Toys Export Catalogue: 23: 20 Ball Adult Puzzle.\*  
Hong Kong Toys Feb. 1995, p. 318: balls inside basket and golf type ball at top of page; smooth tennis type ball under lower bat.\*

(List continued on next page.)

*Primary Examiner*—Stella Reid

*Assistant Examiner*—C. Tuttle

(74) *Attorney, Agent, or Firm*—Cahn & Samuels, LLP

(57) **CLAIM**

The ornamental design for a combined baseball stacking puzzle and pieces, as shown and described.

**DESCRIPTION**

FIG. 1 is a perspective view of my new design for a combined baseball stacking puzzle and pieces in its assembled state;

FIG. 2 is a perspective photographic view shown without the cover for a clear disclosure of the stacked puzzle pieces; FIG. 3 is a top view of FIG. 1.

FIG. 4 is a bottom view with the base removed to show the undersides of the assembled puzzle pieces;

FIG. 5 is a perspective view showing the base and piece elements of the design in a partially assembled state;

FIG. 6 is a top perspective view of the base portion of the design;

FIG. 7 is a bottom view of cover portion of the design;

FIG. 8 is a bottom and side perspective view of the cover portion of the design;

FIG. 9 is a perspective view showing the puzzle piece elements of the design in an unassembled state;

FIG. 10 is a top view showing of one of the puzzle piece elements separately for clarity of disclosure; and,

FIG. 11 is a top view showing a pair of the puzzle piece elements in a stacked relationship.

**1 Claim, 5 Drawing Sheets**

